Step II :

```
       3
  3 | 9 98 56
    |-9 ↓
  --+------
    | 0 98
```

Step III :

```
       3_
  3 | 9 98 56
    |-9
  --+------
 6_ | 0 98
```

Step IV :

```
       3 1
  3 | 9 98 56
    |-9
  --+--------
 61 | 0 98
    |-  61 ↓
    |  ----
    |  3756
```

Step V :

```
        316
  3  | 9 98 56
     |-9
 ----+--------
 61  | 0 98
     |-  61 ↓
     |  -----
 626 |  3756
     |- 3756
     |  -----
     |     0
```

$\therefore \sqrt{99856} = 316$

Now, let us try to understand long division method of square roots by some more examples.

Example 7 : Find the square root of 4401604.

Solution :

```
          2098
   2  | 4 40 16 04
      |-4
   40 |   40
      | - 00
  409 |   4016
      | - 3681
 4188 |   33504
      | - 33504
      |   -----
      |       0
```

$\therefore \sqrt{4401604} = 2098$

Example 8 : Find the square root of 1734489 by the long division method.

Solution : Apply long division method to find square root of 1734489.

```
            1317
     1   |  1 73 44 89
         | -1
    23   |  073
         | - 69
   261   |   444
         |  - 261
  2627   |   18389
         |  -18389
             0
```

$\therefore \quad \sqrt{1734489} = 1317$

Example 9 : Find the least number which must be subtracted from 7581 to obtain a perfect square. Find this perfect square and its square root.

Solution :

```
          87
    8  |  75 81
       | -64
  167  |  1181
       | -1169
          12
```

$\therefore$ 12 should be subtracted from 7581 to make it a perfect square.

Hence, the perfect square = 7581 – 12 = 7569

and $\sqrt{7569} = 87$

Example 10 : What least number must be added to 5607 to make the sum a perfect square? Find the perfect square and its square root.

Solution : Try to find the square root of 5607.

```
          74              |            75
    7  |  56 07           |      7  |  56 07
       | -49              |         | -49
  144  |  707             |    145  |  707
       | -576             |         | -725
          131             |            18
```

We observe that $(74)^2 < 5607 < (75)^2$

$\therefore$ 5607 is (725 – 707) = 18 less than $(75)^2$.

So, we must add 18 to 5607 to make it a perfect square.

Hence, the perfect square = 5607 + 18 = 5625

and $\sqrt{5625} = 75$

1. Find the square root of the following numbers by long division method:

(i) 9801 (ii) 6561 (iii) 390625 (iv) 108241

(v) 363609 (vi) 120409 (vii) 1471369 (viii) 57121

(ix) 64432729 (x) 9653449

2. Find the least number which must be subtracted from 6203 to obtain a perfect square. Also, find square root of the number so obtained.
3. Find the greatest number of six digits which is a perfect square. Find the square root of this number.
4. Find the least number which must be added to 6203 to obtain a perfect square. Also, find the square root of the number so obtained.
5. Find the least number of six digits which is a perfect square. Find the square root of this number.

1.4 SQUARE ROOT OF A RATIONAL NUMBER

In this unit, we shall be taking some examples to understand the rules of finding the square root of rational numbers.

Example 11 : Find $\sqrt{49 \times 25}$ and show that it is equal to $\sqrt{49} \times \sqrt{25}$.

Solution :

$$\sqrt{49 \times 25} = \sqrt{7^2 \times 5^2}$$

$$= \sqrt{(7 \times 5)^2}$$

{We know $a^m \times b^m = (ab)^m$}

$$= \sqrt{(35)^2} = 35$$

$$= 7 \times 5$$

$$= \sqrt{49} \times \sqrt{25}$$

Rule I : For perfect squares a and b, $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$.

Example 12 : Consider $\sqrt{\frac{49}{25}}$ and $\frac{\sqrt{49}}{\sqrt{25}}$ and find out whether they are equal.

Solution :

$$\sqrt{\frac{49}{25}} = \sqrt{\frac{7^2}{5^2}}$$

$$= \sqrt{\left(\frac{7}{5}\right)^2}$$

$$\left\{\text{We know } \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m},\ b \neq 0\right\}$$

$$= \frac{7}{5}$$

Also, $\frac{\sqrt{49}}{\sqrt{25}} = \frac{\sqrt{7^2}}{\sqrt{5^2}}$

$$= \frac{7}{5}$$

Thus, $\sqrt{\frac{49}{25}} = \frac{7}{5} = \frac{\sqrt{49}}{\sqrt{25}}$

Rule II : For perfect squares a and b where $b \neq 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Now, let us apply Rule I and II and solve some examples.

Example 13 : Find the value of $\frac{\sqrt{243}}{\sqrt{867}}$.

Solution :

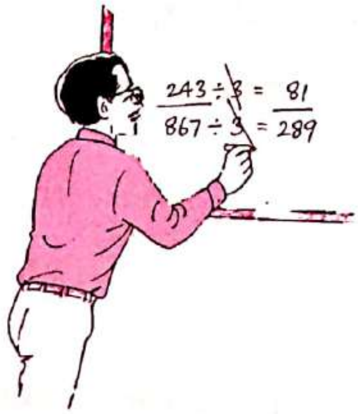
$\frac{\sqrt{243}}{\sqrt{867}} = \sqrt{\frac{243}{867}}$ (using Rule II)

$= \sqrt{\frac{81}{289}}$ (cancel the common factor 3)

$$= \frac{\sqrt{81}}{\sqrt{289}}$$

$$= \frac{\sqrt{9^2}}{\sqrt{17^2}} = \frac{9}{17}$$

Thus, the value of $\frac{\sqrt{243}}{\sqrt{867}} = \frac{9}{17}$.



Example 14 : Find the square root of

(i) $1\frac{56}{169}$ (ii) 14400

Solution: (i) $\sqrt{1\frac{56}{169}} = \sqrt{\frac{225}{169}}$

$$= \frac{\sqrt{225}}{\sqrt{169}} = \frac{\sqrt{15^2}}{\sqrt{13^2}}$$ (by Rule II)

$$= \frac{15}{13}$$

$$= 1\frac{2}{13}$$

(ii) $\sqrt{14400} = \sqrt{144 \times 100}$

$$= \sqrt{144} \times \sqrt{100}$$ (by Rule I)

$$= \sqrt{12^2} \times \sqrt{10^2}$$

$$= 12 \times 10$$

$$= 120$$

1.5 SQUARES OF TERMINATING DECIMALS

Observe some squares

$4^2 = 16$	and	$(0.4)^2 = 0.16$
$12^2 = 144$	and	$(1.2)^2 = 1.44$
$15^2 = 225$	and	$(0.15)^2 = 0.0225$
$18^2 = 324$	and	$(0.18)^2 = 0.0324$

From the above examples, we observe that the square of a decimal consists of twice as many decimal places as given in the number.

Example 15 : Find the square of

(i) 0.41 (ii) 2.6 (iii) 0.25 (iv) 0.009

Solution : (i) $(0.41)^2 = 0.1681$

(ii) $(2.6)^2 = 6.76$

(iii) $(0.25)^2 = 0.0625$

(iv) $(0.009)^2 = 0.000081$

Square Roots of Numbers in Decimal Form (which are perfect squares)

Let us find the square root of a decimal number.

Example 16 : Find the square root of 147.1369.

Solution :

```
            12.13
      1 | 1 47 . 13 69
        |-1
     22 | 0 47
        | -44
    241 |    313
        |   -241
   2423 |    7269
        |   -7269
              0
```

Therefore, $\sqrt{147.1369} = 12.13$

From the above example, the steps of finding out square roots of numbers in decimal form are clear.

Step I : In the integral part, make pairs from the right. But in the decimal part, make pairs from the left.

Step II : Then, find square root as in the case of long division method.

Step III : Place the decimal point as soon as the integral part comes to an end.

Observe that above steps are taken in the following example also.

Example 17 : Find the square root of 0.00059049.

Solution :

```
         0.0243
      ___________________
  0  |  0.00 05 90 49
     | -00
      ___________________
  2  |     05
     |   - 4
      ___________________
  44 |     190
     |    -176
      ___________________
 483 |     1449
     |    -1449
          -----
             0
```

$\therefore \quad \sqrt{0.00059049} = 0.0243$

1.6 APPROXIMATE VALUE OF THE SQUARE ROOTS OF NATURAL NUMBERS (which are not perfect squares)

Long division method is also used to find approximate square roots of numbers or decimals upto certain decimal places. Let us look at the following examples:

Example 18 : Find the square root of 3 upto three decimal places.

Solution : To find the number upto three decimal places which is equal to $\sqrt{3}$, we add 3 pairs of zeros (six zeros) to the right of decimal point.

```
          1.732
       ______________
    1 |  3. 00 00 00
      | -1
       ______________
   27 |  200
      | -189
       ______________
  343 |   1100
      |  -1029
       ______________
 3462 |     7100
      |    -6924
           -----
             176
```

Hence, $\sqrt{3} = 1.732$ upto three decimal places.

Example 19 : Find the square root of $2\frac{1}{5}$, correct to two places of decimal.

Solution :

$$2\frac{1}{5} = \frac{11}{5} = 2.2$$

$$\sqrt{2\frac{1}{5}} = \sqrt{2.2}$$

```
          1.483
     1 | 2. 20 00 00
       | -1
    24 |  120
       | - 96
   288 |  2400
       | -2304
  2963 |   9600
       |  -8889
            711
```

$$\therefore \quad \sqrt{2\frac{1}{5}} = \sqrt{2.2} = 1.483$$

$\simeq$ 1.48 (correct to two places of decimal)

Note: We were required to find the square root of $2\frac{1}{5}$ correct upto two places of decimal. Here, we have found the square root upto three places of decimal. In the third place, we have 3(<5) and therefore, in the final result, 3 is ignored.

Square Root of Other Numbers (not perfect squares) by Estimation

It is easy to work out the square root of a perfect square, but it is really hard to work out the square root of other numbers.

For example, what is the square root of 10?

Well, in such cases, we need to **estimate** the square root.

Now, the perfect squares near to 10 are 9 and 16,

i.e. $9 < 10 < 16$ or

$3^2 < 10 < 4^2$

So we can guess that the answer is between 3 and 4.

Let us try with 3.5 as

$$3 < 3.5 < 4$$

But $\quad 3.5 \times 3.5 = 12.25 > 10,$

i.e. $\quad 3^2 < 10 < (3.5)^2$

Let us further reduce the number 3.5 to 3.2

So $\quad 3.2 \times 3.2 = 10.24 > 10$

Let us try with 3.1 so that

$$3.1 \times 3.1 = 9.61$$

So we can say $\quad 9.61 < 10 < 10.24$

or $\quad (3.1)^2 < 10 < (3.2)^2$

But 10.24 is much closer to 10 as compared to 9.61.

So we can say $\sqrt{10}$ is 3.2 approximately.

1. Find the square root of the following fractions:
   (i) $\frac{324}{361}$ (ii) $\frac{441}{961}$ (iii) $5\frac{19}{25}$ (iv) $21\frac{51}{169}$
   (v) $\frac{5625}{441}$ (vi) $7\frac{18}{49}$ (vii) $23\frac{394}{729}$ (viii) $35\frac{85}{1444}$

2. Find the square root of the following decimal numbers:
   (i) 0.0009 (ii) 0.0081 (iii) 0.012321 (iv) 7.29

3. Find the square root of:
   (i) 0.053361 (ii) 0.00053361 (iii) 150.0625 (iv) 0.374544
   (v) 610.09

4. Find the square root of the following (correct to three decimal places):

(i) 7 (ii) 2.5 (iii) $2\frac{1}{12}$ (iv) $367\frac{2}{7}$

5. Estimate the value of the following to the nearest to one decimal place.

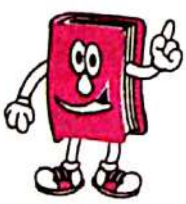
(i) $\sqrt{90}$ (ii) $\sqrt{150}$ (iii) $\sqrt{600}$ (iv) $\sqrt{1000}$

6. Devika has a square piece of cloth of area 150 m^2 and she wants to make 10 square-shaped scarves of equal size out of it. What should possibly be the length of the side of the scarf that can be made out of this piece? (Use the method of estimation in finding the length of the scarf.)

7. The area of a square plot is 800 m^2. Find the estimated length of the side of the plot.

BRAIN TEASERS

1. Find the square root of 10, correct to four places of decimal.
2. Find the values of $\sqrt{3.1428}$ and $\sqrt{0.31428}$.
3. Simplify: (i) $\frac{\sqrt{0.0441}}{\sqrt{0.000441}}$ (ii) $\sqrt{49}+\sqrt{0.49}+\sqrt{0.0049}$
4. The area of a square field is $101\frac{1}{400}$ m^2. Find the length of one side of the field.
5. What is that decimal which when multiplied by itself gives 227.798649?
6. 8649 students are sitting in a lecture hall in such a manner that there are as many students in a row as there are rows in the lecture hall. How many students are there in each row of a lecture hall?
7. A General wishing to draw up his 64,019 men in the form of a square found that he had 10 men extra. Find the number of men in the front row.



YOU MUST KNOW. . .

1. The square of a number is that number raised to the power 2.
2. A perfect square number is never negative.
3. A number ending in 2, 3, 7 or 8 is never a perfect square.

4. (i) Squares of even numbers are even.

   (ii) Squares of odd numbers are odd.

5. A perfect square number leaves a remainder 0 or 1 on division by 3.

6. There are no natural numbers p and q such that $p^2 = 2q^2$.

7. If a and b are perfect squares ($b \neq 0$), then

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$

and

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}.$$

8. The square root of a perfect square can be obtained

   (i) by finding prime factors.

   (ii) by long division method.

9. The pairing of numbers in the division method starts from the decimal point. For the integral part, it goes from right to left and for the decimal part, it goes from left to right.

10. If p and q are not perfect squares, then to find $\sqrt{\frac{p}{q}}$, we express $\frac{p}{q}$ as a decimal and then apply division method.